ALGORITHMS FOR THE INTERSECTION OF COMPLEX MARITIME BOUNDARIES

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ABSTRACT

The maritime tenements established for oil-search and recovery operations are invariably defined as areas on the surface of a reference ellipsoid with boundaries made up of meridians, parallels of latitudes, loxodromes (lines of constant azimuth) or geodesics. Typical problems presented to hydrographic surveyors working in this field are the establishment of the coordinates of intersections of boundaries, the spatial relationships between facilities and tenement boundaries and the determination of tenement areas. While the process of computing a string of ellipsoidal coordinates that define the plane curve between two points is widely known, that for computing similar strings for geodesics and loxodromes is less so. With GPS offering precise fixation at sea, the assumption that for all practical purposes plane curves and geodesics are the same, is now being challenged, making it necessary to treat geodesics in a more rigorous manner. This paper reports on the development of a software package for computations to precisely establish maritime boundaries defined as geodesics or loxodromes.

INTRODUCTION

1. The computational problem

Maritime boundaries defining Exploration Development Titles or Tenements are made up of a combination of parallels of latitude, meridians, loxodromes and geodesics. Typical configurations can be seen in Figures 1 and 2.

Figure 1 shows the Zone of Cooperation between Indonesia and Australia and its relationship to the sea-bed boundary and the extent of the Australian Exclusive Economic Zone (AEEZ). The sea-bed boundary is defined by a series of loxodromes. The Zone of Cooperation is divided into 3 sections for which differing levels of cooperation have been agreed. The boundaries defining the total Zone of Cooperation and those delineating the sections are geodesics. That between the two most southern sections is coincident with the boundary of the AEEZ.

The Exploration Development Titles within the central Zone of Cooperation are shown in Figure 2. These are defined on a graticule of meridians and parallels of latitudes.

The principal problem is to establish a generic algorithm to determine the geometrical relationships between any combination of the 4 types of boundary. In brief, this includes the computation of the coordinates of the intersection of any two boundaries, the areas of tenements, the separation between a geodesic and loxodrome and the spatial relationship between boundaries and discrete points within their vicinity.

Examples of intersections that can been seen in Figure 2 include:

- the sea-bed boundary (loxodrome) with the boundary of the Zone of Cooperation (geodesic),
- the Exploration Development Title boundaries (meridian or parallel) with those of the Zone of Cooperation (geodesic), and
- those between title boundaries made up of meridians and parallels.

When geodesics and loxodromes are involved, these computations can be extremely complex if attempted on the surface of the ellipsoid. They can be simplified if undertaken on an appropriate map projection. For example, meridians, parallels and loxodromes appear on the Mercator projection as straight lines. Thus the computation of intersection of any

two of these lines becomes a far simpler problem on the projection than that on the ellipsoid.

Geodesics are curved on the Mercator projection and most practically represented as a string of coordinates connected by straight line segments. The intersection of a geodesic with any other line can then be done by checking each segment for an intersection. Also the separation between geodesic and loxodrome can be easily calculated via the Mercator projection once the string of coordinates has been computed. It will be seen that the use of an equal area projection simplifies the computation of tenement areas.

2. Definitions

A geodesic is the line of the shortest surface distance between two points on a reference ellipsoid. The azimuth varies continuously with distance along the geodesic - the initial azimuth at a terminal can be computed by a correction to the azimuth of the plane curve from that terminal to the other. For geodesics of up to 3000 kilometres the length can be taken as that of the plane curve. Plane curve azimuths and lengths can be computed from geographical coordinates of the terminals using by the classical formulae of Clarke, Robbins or Rudoe (see Bomford, 1962). Alternatively, the azimuth the plane curve can be computed by vector geometry from the geocentric coordinates of the terminals.

There are formulae available in the literature to compute the change in latitude, longitude and azimuth of a geodesic as a function of initial azimuth and length along the geodesic. As shown in §4, these formulae are most useful in computations involving maritime boundaries.

A loxodrome is the line of constant azimuth on the surface of a reference ellipsoid. The line would not be the first choice of many as a maritime boundary but owes its popularity to its convenience in navigation. It appears as a straight line on the Mercator projection and being of constant azimuth, provides a simple path to navigate between two points. Again there are formulae available (see Bowring, 1985) to compute the change in latitude and longitude of a loxodrome as a function of initial azimuth and length along the loxodrome. These are not used in this development as simpler methods are available to generate a series of geographical coordinates along a loxodrome by using the Mercator projection.

Meridians and parallels need no definition but perhaps it should be noted that:

- geodesics and loxodromes coincide along a meridian,
- parallels are loxodromes, but
- a geodesic between two points at the same latitude does not follow the parallel of latitude.

ALGORITHMS

3. Computational Requirements

The algorithms required to solve problems associated with maritime boundaries are not commonly encountered modern geodetic computations. By and large the current approach is to express geodetic measurements, and particularly GPS baselines, as functions of geocentric coordinates without need to reference these to the surface on an ellipsoid. These functions are relatively simple and for all practical purposes, exact. Thus all the fundamental geodetic computations can be done in terms of geocentric coordinates with the reference ellipsoid only coming into play when wishing to transform these into geographical or map grid coordinates.

Maritime boundaries however are defined as curves on the surface of an ellipsoid either in terms of the latitude and longitude of their terminals or as a distance and initial azimuth from a given starting point. While the definition of parallels and meridians on the ellipsoid is straight forward, geodesics and loxodromes are more difficult to deal with. Formulae are available but they are complex, usually being expressed as a mathematical series. They have been developed for moderate distances that are often much shorter than the lengths that need to be considered for large oil tenements and are when used over these longer distances become inaccurate.

Prior to GPS the fact that the definition of maritime boundaries was of limited accuracy was of little consequence as the precision of positioning was low. As hydrographic surveyors now have the ability to locate craft and structures many hundreds of kilometres offshore to a high level of accuracy, the definition of the maritime boundaries needs to be compatible to this positioning capability.

Algorithms required to support the treatment of complex maritime boundaries can be considered under the following headings:

- the computation of the initial bearing and length of a geodesic or loxodrome defined by the geographical coordinates of its terminals,

- the computation of the geographical coordinates of the point of intersection of any two tenement boundaries which may be defined as a meridian, parallel, geodesic or loxodrome,

- the computation of the geographical coordinates of a point on a geodesic or loxodrome at a given distance from a terminal,

- the generation of a series of geographical coordinates at appropriate spacing to serve as a the "trace" of a geodesic or loxodrome on the ellipsoid,

- the determination of the spatial relationship of a point of given latitude and longitude to a meridian, parallel, geodesic or loxodrome - for example the coordinates of the point on a boundary closest to a given latitude and longitude,

- the area of tenements bounded by meridians, parallels, geodesics or loxodromes, and

- the area and separation between the loxodrome and geodesic which run between any two nominated points.

4. Computation of intersections of maritime boundaries

The computation of the intersection of say a loxodrome and geodesic on the surface of the ellipsoid can be done as an iterative process using the formulae mentioned in §(2). However, as the loxodrome, meridian and parallel appear as straight lines on the Mercator projection, the intersection problem can be done more simply if done on that surface. The transformation formulae from geographical coordinates (ϕ, λ) to projection coordinates are simple and can be expressed as:

$$E = a(\lambda - \lambda_0)$$
$$N = M(\phi)$$

where,

$M(\phi)$	~	is the meridian distance of the latitude ϕ,
λ_0	~	the longitude of the meridian adopted as origin for the eastings,
a	~	the semi-major axis of the ellipsoid,
E	~	easting
N	~	northing

The expression for the meridian distance is given in Bomford(1962) or National Mapping Council of Australia (1986).

Meridians, parallels and loxodromes can be defined on the projection by merely transforming the geographical coordinates of their terminals into projection coordinates. The geodesic however will appear as a curved line on the Mercator projection. Thus it needs to be defined as a series of points at an appropriate separation. This is done by firstly generating a series of geographical coordinates on the ellipsoid (using the formulae mentioned in §(2)) and then transforming these to projection coordinates. Figure 3 shows what might be a typical tenement defined by intersections of a segmented geodesic, meridian, parallel and loxodrome. Any convenient meridian can be chosen for the origin of the easting coordinates - perhaps choosing one a little to the west of the most westerly point in the tenement so that all easting coordinates, at least, are positive.

The intersections between two straight lines (represented by lines 1-2 and 3-4 in Figure 4), can be done by solving for the scale factors α and β in the vector equation:

$$\alpha \mathbf{v}_{12} + \beta \mathbf{v}_{43} = \mathbf{v}_{31}$$

where,

$\mathbf{v}_{ij}$	~	is the vector between points i and j.

The coordinates of the intersection point (5) are given by either of the equations:

$$\mathbf{v}_5 = \mathbf{v}_1 + \alpha\mathbf{v}_{12}$$
$$\mathbf{v}_5 = \mathbf{v}_3 - \beta\mathbf{v}_{43}$$

The elements of $\mathbf{v}_5$ are the coordinates of point (5) as $\mathbf{v}_1$, $\mathbf{v}_3$ and $\mathbf{v}_5$ are the position vectors of the associated points.

If α or β are negative or larger than the magnitude of the vector on which they operate, the line segments do not intersect.

When the intersection of segmented lines (geodesics) is being considered, each of the segments of one line needs to be checked with each of the segments of the other until an intersection is found that lies within the two segments being considered. This may mean significant repeated computations before an intersection is found but the algorithm shown above is very efficient as it requires few computer operations. However it helps to avoid excessive computation if the segmenting of a geodesic is initially very course - say in 10-kilometre sections. Once the segment that intersects has been found, this can be re-segmented into smaller sections and the intersection process repeated. This recursive procedure can be continued until the section size is small enough to be considered coincident with the geodesic.

6. Spatial relationships between points and boundaries

The most fundamental spatial relationship which needs to be considered is that between a point at known latitude or longitude and any maritime boundary. The full problem consists of finding:

- the shortest distance between the point and the boundary,
- the point on the boundary where the shortest distance intersects, and
- the azimuth and length of the shortest line.

(Problems involving the intersection of a line through a given point on a particular azimuth, can be treated as an intersection problem discussed in §(5)).

The most difficult case is finding the shortest distance from a point to a curving boundary. The algorithm adopted here is to compute the distances from the nominated point to every point in the string that represents the curving boundary as shown in Figure 5. The minimum sum of any two consecutive distances (l_2+l_3 in the diagram) will locate the section that contains the closest point. The procedure of re-segmenting into smaller sections and recomputing the sub-section containing the closest point can be repeated until the distances from the nominated point to the two ends of the line are not significantly different. At this stage, the midpoint of the segment can be taken as the closest.

If the shortest distance from point to boundary is to be along a geodesic, the algorithm needs be implemented on the surface of the ellipsoid with the distances from nominated point to boundary points being taken as the length of the plane curve and given by Robbins or Rudoe's formula. If the shortest distance is to be along a loxodrome, the computation is done on the Mercator projection.

The algorithm adopted here is more complex than what might appear to be a simpler solution of finding the perpendicular from the nominated point to a boundary segment on the Mercator projection. This solution was not chosen as it gives only the shortest loxodrome from a point to a boundary, but does not treat the shortest geodesic.

One other case that must be considered in finding the shortest distance from a nominated point to a boundary, is when the point falls outside the "square offset range" of a boundary - the shaded areas shown in Figure 6. In this circumstance the shortest distance will be that to the closest terminal of the boundary.

7. Areas

The computation of the areas of polygons on the surface of the ellipsoid is complex. Areas of lunes truncated by parallels of latitude can be determined by a relatively simple integration (see Lauf, 1983). When loxodromes and geodesics at random azimuths become involved, this approach becomes complex but could be used by computing multiple integrations of the narrow lunes formed by small segments of the loxodrome or geodesic. As each lune would be truncated by the parallel at the mid-latitude of the segment, the area would be approximate but by making the segment size appropriately small this inaccuracy could be kept to an insignificant level. This technique would give the area between

the boundary and the pole for each boundary in the tenement - appropriate addition and subtraction of these areas would lead to the area of a polygonal tenement.

A simpler approach, and the one used in this development, is to segment each boundary of a tenement and project the multi-sided polygon so formed onto an equal area projection of the reference ellipsoid. Any equal area projection would suffice - the one chosen here is the cylindrical equal area projection which was developed as a simplification of the Albers Equal Area projection (U.S. Department of Commerce, 1934). It should be noted that on this projection meridians and parallels appear as straight lines but both geodesics and loxodromes are curved.

The transformation formulae are:

$$E = a(\lambda - \lambda_0)$$
$$N = a(1-e^2)(\sin\phi + (2/3)e^2 \sin^3\phi + (3/5)e^4 \sin^5\phi + (4/7)e^6 \sin^7\phi + (5/9)e^8 \sin^9\phi + \ldots$$

where,

e ~ is the eccentricity of the ellipsoid.

Other terms have been defined previously.

The precision of both techniques depends on the section size chosen for the segmenting of each boundary. Investigations are continuing as to what precision is needed and subsequently the segment size to meet that precision.

CONCLUSIONS

As indicated in the paper, computations involving the spatial relationships between points, geodesics and loxodromes can be difficult to establish when working with formulae for these curves on the surface of the ellipsoid. Maritime boundaries are often defined in terms of these lines and with GPS providing hydrographic surveyors with the ability to precisely locate facilities at sea, the computation of boundaries need to be done with equal precision.

The problems likely to be met by surveyors defining maritime boundaries are listed in §(3). A software package has been developed to provide solutions to these in which use has been made of 2 map projections to simplify the computations. The testing completed to date indicates that the algorithms used are well within geodetic accuracy for lines up to 500 kilometres. Further investigations are needed to evaluate their accuracy with lines over 1000 kilometres as there is some evidence that the surface formulae on which they are based break-down over those distances.

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U.S. Department of Commerce. (1934) *Elements of Map Projection.* Special Publication NO. 68 .

Figure 1 Zone of Cooperation Between Indonesia and Australia (not available digitally)

Figure 2 Exploration Development Titles Within the Zone of Cooperation (not available digitally)

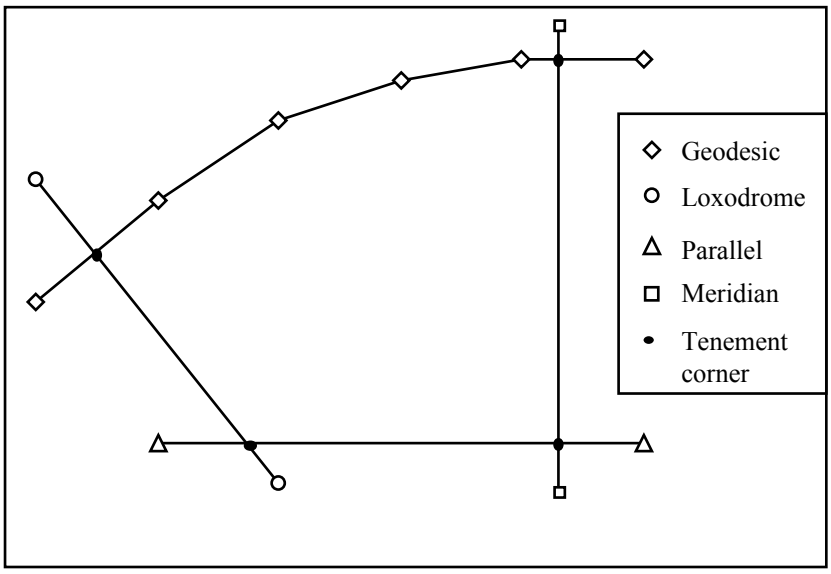

Figure 3 Tenement plotted on Mercator projection and consisting of intersecting meridians, parallels, geodesics and loxodrome

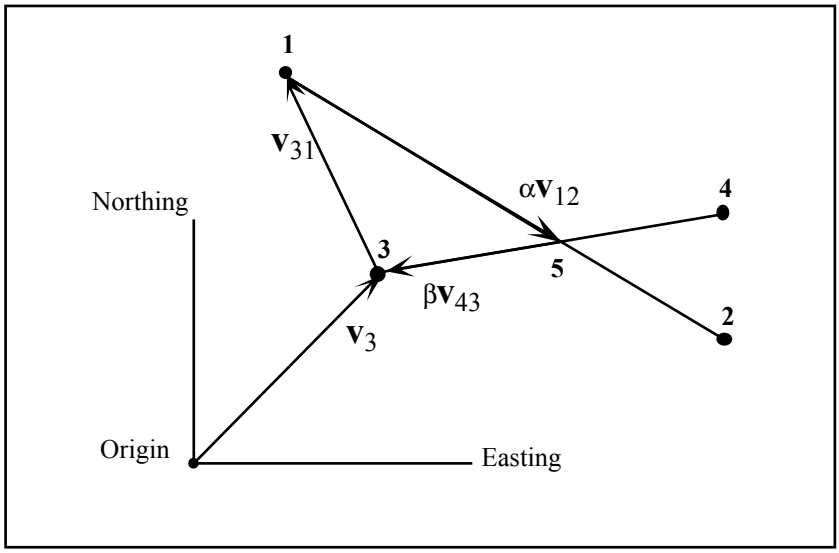

Figure 4 Vector diagram of intersection of boundary line segments

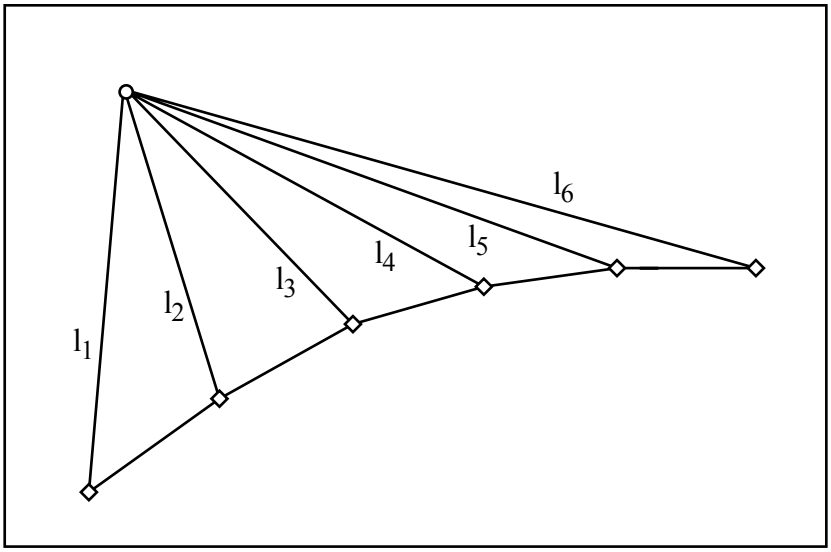

Figure 5 Locating closest segment to point of given latitude and longitude

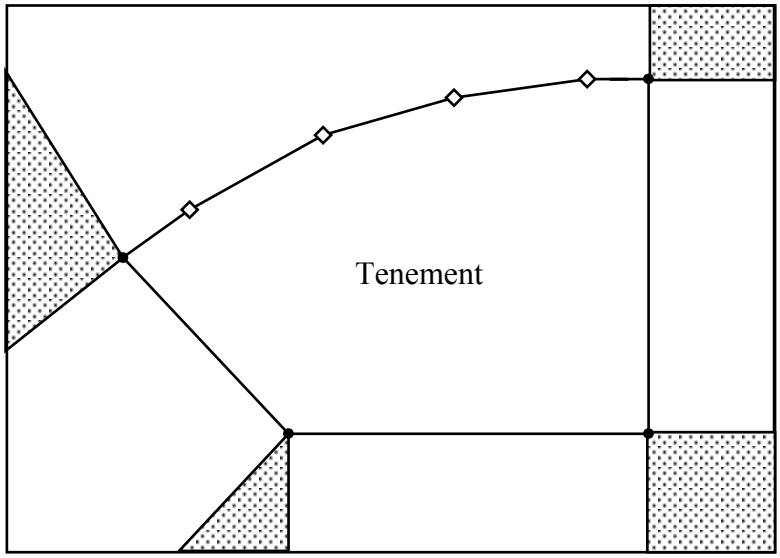

Figure 6 Shaded areas are outside "square offset range" of tenement boundaries